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Solving a Nonlinear Inverse Convection Problem Using the Sequential Gradient Method

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This study investigates a nonlinear inverse convection problem for a laminar-forced convective flow between two parallel plates. The upper plate is exposed to unknown heat flux while the lower plate is insulated. The unknown heat flux is determined using temperature measured on the lower plate. The thermophysical properties of the fluid are temperature dependent, which renders the problem nonlinear. The sequential gradient method is applied to this nonlinear inverse problem in order to solve the problem efficiently. The function specification method is incorporated to stabilize the sequential estimation. The corresponding adjoint formalism is provided. Accuracy and stability have been examined for the proposed method with test cases. The tendency of deterministic error is investigated for several parameters. Stable solutions are achieved even with severely impaired measurement data.

Key Words : Inverse convection Problem, Sequential Gradient Method, Function Specification Method, Adjoint Formulation, Sensitivity Problem

Nomenclature

C : Volumetric heat capacity
f : Unknown heat flux
$\tilde{f}$: Unknown heat flux with function specification
H : Channel height
J : Residual norm
k : Thermal conductivity
L : Channel length
m : Number of future time steps
M : Number of time steps
N : Number of nodal points along duct
p : Parameter for iterative improvement
Pe : Peclet number
R : The total number of repeated estimations using the iterated final condition
S : Conjugate direction
t : Time

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t_f : Final time
t_l : Final time for each sequence
t_o : Initial time for each sequence
T : Temperature
T_o : Initial temperature
T_{in} : Inlet temperature
u : Velocity, or step function
U : Mean velocity
v : Perturbed temperature
x : Axial distance along channel
y : Transverse coordinate
Y : Measured temperature

Greek

β : Step size
γ : Conjugate coefficient
Δt : Time step
Δx : Distance between nodal points along x-direction
σ : Standard deviation
τ : Time variable for solution of adjoint problem

Subscript

i : Index

o : Nominal value
$+$: Dimensionless variable

1. Introduction

The inverse analysis facilitates the estimation of imperfectly described boundary conditions in heat transfer problems, from the temperature measurement inside or on the boundary. A variety of studies has been performed for such problems, called inverse heat conduction problem, considering heat conduction of solid media. Comprehensive reviews can be found regarding such problem in some literatures (Beck et al., 1985 and Alifanov, 1994). On the other hand, a different kind of inverse problem arises when the unknown boundary condition for a convective heat transfer problem is presented. It is called an inverse convection problem. While inverse heat conduction problems have been studied extensively, inverse convection problems have received less attention.

The convective flow inside a duct is of interest, since it can be applied to a variety of heat exchanging systems. In particular, characterizing the effects of stationary or transient heat flux applied on the duct wall is important in the control of such system. Thus, the direct problem of the above described convection problem has been investigated for various boundary conditions and duct shapes in great detail (Shah and London, 1978). Several researchers have carried out the inverse estimation of wall heat flux on a duct wall, which is a characteristic inverse convection problem. Moutsoglou conducted the inverse analysis for determining the wall heat flux in a vertical channel with laminar free-convective flow using the sequential function specification method (Moutsoglou, 1989). Huang and Özişik estimated the stationary wall heat flux of laminar channel flow using the conjugate gradient method (Huang and Özişik, 1992). Liu and Özişik estimated the transient wall heat flux for the turbulent channel flow (Liu and Özişik, 1996). Li and Yan investigated a similar inverse problem for the laminar flow (Li and Yan, 1999). Park and Lee have solved an inverse convection problem based on the Karhunen-Loeve Galerkin procedure (Park and Lee, 1998).

A forced laminar convective flow between two parallel palates is considered. This work aims to estimate the unknown heat flux applied to one of the plates. Meanwhile, an adiabatic condition is imposed on the other plates. Convection problems with these boundary conditions have been investigated comprehensively since such asymmetric heat exchanges occur frequently in thermal systems and experimental setups (Park and Lee, 1998, Chin et al., 1988, Makkawi, et al., 1998, Tao, 1961). In this study, the temperature readings are obtained on the adiabatic wall whereas previous studies considers inside measurements (Liu and Özişik, 1996, Park and Lee, 1998). The acquisition of temperature inside a duct or on a heated surface is not an easy task (McGee, 1988). Furthermore, temperature readings on an insulated wall can be conditioned well, compared to those measured in a flow or on a heat-exchanging surface.

The sequential gradient method, whose validity is examined in the previous works (Reinhart and Hao, 1996, Dowding and Beck, 1999), combined with the function specification method (Beck et al., 1985) facilitates solving this inverse convection problem. The temperature dependent thermophysical properties are utilized in this analysis, which makes it a nonlinear problem. The corresponding adjoint formulation is provided for the gradient evaluation (Alifanov, 1994, Liu and Özişik, 1996). A sequential version of the gradient method, which considers a constant function specification over a given interval and retention of one time step per interval, is implemented. Then, the developed method is applied to several test cases. The tests have been performed for varying flow rate, geometry, and error level.

In summary, this study presents a solution method for nonlinear inverse convection problem utilizing the sequential gradient method, which is rather recently developed, and applies the method to several test problems.

2. Problem Statement and Formulation

A fully-developed forced laminar convection between two parallel plates is investigated. The considered geometry is illustrated in Fig. 1. While the lower plate is insulated, the upper plate is exposed to the unknown heat flux varying in time and space, with known inlet and initial temperature. The measurement data are obtained at N equi-spaced locations on the outside of the lower plate.

In order to alleviate the nonlinear nature of the estimation and improve the computational efficiency, the sequential gradient method is incorporated with the function specification method. This work selects the constant function specification for its stability and simplicity. A corresponding adjoint formulation for the nonlinear inverse convection problem is made (Alifanov, 1994). The formulation involves the direct problem, the sensitivity problem, and the adjoint problem. They are presented below.

2.1 Direct problem

Neglecting the axial diffusion, the energy equation for a laminar forced convective flow between two parallel plates, as shown in Fig. 1, is given by

$$C\frac{\partial T}{\partial t}+Cu\frac{\partial T}{\partial x}=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right),\ \text{in } 0<x<L,\ 0<y<H \quad (1a)$$

where, the fully developed laminar velocity profile is given by

$$u(y)=6U\left(\frac{y}{H}-\frac{y^2}{H^2}\right) \quad (1b)$$

where U is the mean velocity and H is the distance between upper and lower plate. The initial condition and boundary conditions are

$$T(x,\ y,\ t_o)=T_o \quad (1c)$$

$$T(0,\ y,\ t)=T_{in} \quad (1d)$$

$$\frac{T(x,\ 0,\ t)}{\partial y}=0 \quad (1e)$$

$$k\frac{T(x,\ H,\ t)}{\partial y}=f(x,\ t) \quad (1f)$$

where t_o is the initial time, T_o is the initial temperature, and T_{in} is the inlet temperature.

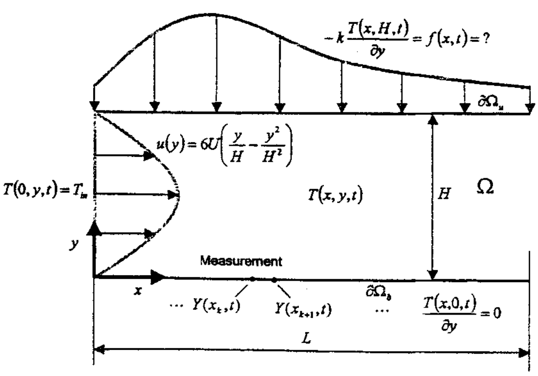

Fig. 1 Overview of the inverse convection problem considered

2.2 Sensitivity problem

When the unknown f is perturbed by a small amount Δf, T undergoes a perturbation $T+v$. Then, substituting in the original governing equations T by $T+v$, f by Δf; perturbed equations are obtained. Then, subtracting the original equations from the perturbed equations and neglecting the terms of the second order of smallness give the following sensitivity equations.

$$\frac{\partial Cv}{\partial t}+u\frac{\partial Cv}{\partial x}=\frac{\partial^2(kv)}{\partial y^2} \quad (2a)$$

$$v(x,\ y,\ t_o)=0 \quad (2b)$$

$$v(0,\ y,\ t)=0 \quad (2c)$$

$$\frac{v(x,\ 0,\ t)}{\partial y}=0 \quad (2d)$$

$$\frac{\partial kv(x,\ H,\ t)}{\partial y}=\Delta f(x,\ t) \quad (2e)$$

2.2 Sequential estimation

The current inverse estimation is implemented in a sequential manner. This method is similar to the conventional sequential function specification method (Beck et al., 1985) except that the adjoint formalism (Alifanov, 1994) is utilized instead of the least squares method utilizing sensitivity coefficients.

Sequential schemes for the gradient method are already described in the previous works (Reinhart and Hao, 1996, Dowding and Beck, 1999). However, the sequential procedure is briefly described again here. The whole time domain $0<t\leq t_f$ is discretized to M uniform intervals with a fixed time step $\Delta t\equiv t_f/M$. Every sequence considers the time interval $t_o<t\leq t_l$. Here, t_o is the latest time when $f(x, t)$ is regarded as known, and t_l is given by $t_l\equiv t_o+m\Delta t$ where m is the number of future time steps. Typical sequential methods retain only the first time step for every sequential interval (Beck et al., 1985). Retaining more than one time step is investigated in the previous works (Dowding and Beck, 1999), but it requires complicated statistical analysis to be reliable (Flach and Özişik, 1995). Thus, this study considers retaining the first time step only. Hence, the time interval is shifted by one time step after each sequential estimation.

2.4 Inverse problem and adjoint formulation

Since the inverse problem is mathematically ill-posed (Beck et al., 1985 and Alifanov, 1994), it is to be treated as an optimization problem (Jarny, et al., 1991). The optimization problem aims to achieve statistical consistency between the measured and computed temperatures. The statist -cal consistency can be measured by the following residual functional for single sequence.

$$J=\int_{t_o}^{t_l}\int_0^L[T(x, 0, t)-Y(x, t)]^2dxdt \tag{3}$$

where Y is the measured temperature. Assuming the constant standard deviation of the measured temperature over the sequential interval and space, the admissible target value of the residual functional becomes

$$J\leq L(t_l-t_o)\sigma^2 \tag{4}$$

where σ is the standard deviation of the measured temperature. In order to achieve the above optimality, the gradient evaluation is required. The gradient of J can be obtained by utilizing the solution of the adjoint problem. Let us introduce Lagrange multiplier λ to derive the adjoint problem. The Lagrangian function can be written as follows by the modifying Eq. (3) to absorb Eq. (1a) as a constraint. Integrating Eq. (1a) multiplied by λ over time and space domain, and then adding this expression to the residual functional, gives

$$\begin{aligned}J=&\int_{t_o}^{t_l}\int_0^L[T(x, 0, t)-Y(x, t)]^2dxdt\\&+\int_{t_o}^{t_l}\int_0^H\int_0^L\lambda\left[\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-C\frac{\partial T}{\partial t}-Cu\frac{\partial T}{\partial x}\right]dxdydt\end{aligned} \tag{5}$$

Deliberation on the increment $\Delta J\equiv J(f+\Delta f)-J(f)$ and neglecting the member of the second order of smallness, we obtain the following variation of the residual J.

$$\begin{aligned}J=&\int_{t_o}^{t_l}\int_0^L 2[T(x, 0, t)-Y(x, t)]v(x, 0, t)\,dxdt\\&+\int_{t_o}^{t_l}\int_0^H\int_0^L\lambda\left[\frac{\partial^2 kv}{\partial y^2}-\frac{\partial Cv}{\partial t}-u\frac{\partial Cv}{\partial x}\right]dxdydt\end{aligned} \tag{6}$$

Subsequent to integrating by parts, utilizing the arbitrariness of the increments of state variable v and the boundary conditions of the sensitivity problem, the following adjoint problem is obtained.

$$-C\frac{\partial\lambda}{\partial t}-uC\frac{\partial\lambda}{\partial x}=k\frac{\partial^2\lambda}{\partial y^2} \tag{7a}$$

$$\lambda(x, y, t_l)=0 \tag{7b}$$

$$\lambda(L, y, t)=0 \tag{7c}$$

$$\frac{\lambda(x, 0, t)}{\partial y}=-2[T(x, 0, t)-Y(x, t)] \tag{7d}$$

$$\frac{\partial\lambda(x, H, t)}{\partial y}=0 \tag{7e}$$

Via the substitutions that $\xi=L-x$ and $\tau=t_l-t$, the adjoint problem can be solved by a similar way as the direct and sensitivity problems are solved. Then, a constant function $\bar{f}$ is specified over the entire interval $t_o<t\leq t_l$, which is expressed as

$$\bar{f}(x)=f(x, t) \tag{8}$$

Then, using the identity $\Delta\bar{f}(x)=\Delta f(x, t)$ in Eq. (2e), ΔJ becomes

$$\Delta J=-\int_{t_o}^{t_l}\int_0^L\lambda\Delta\bar{f}dxdt \tag{9}$$

By definition of the square-integrable function space, the following expression is valid (Alifanov, 1985).

$$\Delta J=\int_0^L J'(x)\Delta \tilde{f}dx \tag{10}$$

Comparing eqs. (9) and (10) gives the following gradient equation.

$$J'(x)=-\int_{t_0}^{t_f}\lambda(x,\ H,\ t)\,dt \tag{11}$$

2.5 Optimization procedure

The iterative method using the conjugate gradient method already established in the previous investigation is utilized (Alifanov, 1985). The unknown heat flux is updated by

$$f=f^p-\beta S \tag{12}$$

The conjugate direction is of the form

$$S=J'+\gamma S^p \tag{13}$$

Here, the conjugate coefficient γ is set as 0 for initial state, otherwise is given by the following expression, which is suitable for nonlinear IHCPs (Alifanov, 1985 and Jarny et al., 1991).

$$\gamma=\left(\int_0^L J'(x)\,[J'(x)-J'^p(x)]\,dx\right)\Big/\left(\int_0^L[J'^p(x,\ t)]^2dx\right) \tag{14}$$

Furthermore, β can be determined by linearization of $J(f^p-\beta S)$ followed by setting $\partial J/\partial\beta=0$. It follows that

$$\beta=\left(\int_{t_0}^{t_f}\int_0^L[T(x,0,t)-Y(x,t)]\,v(x,0,t)\,dxdt\right)\Big/\left(\int_{t_0}^{t_f}\int_0^L[v(x,0,t)]^2dxdt\right) \tag{15}$$

When the selected β incurs the increase of J, which can occur due to the augmented nonlinearity caused by the temperature dependence of the thermo physical properties, the golden section search facilitates the correction of β.

In summary, the solution procedure is as follows:

1. Set m, M.
2. Give an initial guess for f within the given interval using the result from previous interval.
3. Solve the direct problem given by Eq. (1) to update the temperature field.
4. Check Eq. (4). If satisfied go to 10.
5. Solve the adjoint problem given by Eq. (7).
6. Calculate the gradient using Eq. (11) to obtain λ.
7. Evaluate S using Eqs. (13) ~ (14).
8. Solve the sensitivity problem using Eq. (2) to obtain $v(x,\ y,\ z,\ t)$ after setting $\Delta f=S$.
9. Calculate β using Eq. (15) and update f using Eq. (12) and go to 3.
10. Record $f(x,\ y,\ t_o+\Delta t)$ as heat flux for the current time and set $t_o=t_o+\Delta t$.
11. If $t_o>t_f$, finish, else go to 2.

2.6 Numerical discretization

Numerical solutions of the direct, sensitivity and adjoint problems are essential in this inverse estimation. This work employs the finite volume method with the upwind scheme in x-direction and the Crank-Nicolson scheme in the time domain (Patankar, 1980). The whole space domain is discretized by a $N\times N$ uniform grid system.

2.7 Singularity in the final step

Inherently, the adjoint formulation for the current inverse problem can involve singularity in gradient due to the boundary and initial conditions for the adjoint equations (Eqs. (7b) and (7c)). Thus, $f(x^+,\ t_f)$ and $f(L,\ t^+)$ cannot change its value while the optimization procedure with the conventional gradient method. This singularity cannot be avoided without additional modification in the conventional gradient method. However, there is no such singularity in the time domain with the current method thanks to sequential estimation since the gradient is averaged over the sequential interval (Reinhart and Hao, 1996).

Despite the absence of singularity in the time domain, we still have a singularity in the space domain at $x=L$. The best way is to perform the inverse estimation with known $f(L,\ t^+)$ (for example, adiabatic condition). However, sometimes $f(L,\ t^+)$ cannot be known prior to estimation and the singularity should be resolved. In order to overcome this singularity, several methods have been proposed (Alifanov, 1995). The simplest technique is to utilize the iterated final condition (Silva and Özişik, 1992). This method repeats the estimation by taking the initial guess for $f(x^+, t^+)$ as the value from the previous estimation $f^p(x_e, t^+)$. Here x_e is selected near $x=L$ by determining a proper value of p in the following expression.

$$x_e = L - p\Delta x \tag{16}$$

where

$$\Delta x = L/(N-1) \tag{17}$$

Furthermore, the total number of repeated estimations using the iterated final condition (the number of repeating the whole solution procedure described above) is set as R. This method is very easy to implement with slight modifications to the regular method. In addition, a few more methods, which are more systematic, can be exploited, but they are complicated and can sometimes cause more resolution loss than the described method due to integration (Silva and Özişik, 1992) or regularization (Dowding and Beck, 1999) especially in the region far from the final point.

3. Results and Discussion

The presented method is tested for a couple of heat flux forms to examine its validity. The test cases consider conditions stated below.

3.1 Dimensionless variables

Let us introduce the following dimensionless variables involving the Peclet number, Pe_H,

$$Pe_H = UHC_o k_o^{-1},\ x^+ = xL^{-1} \tag{18a}$$

$$t^+ = k_o t C_o^{-1} L^{-2},\ f^+ = f f_o^{-1} \tag{18b}$$

$$\sigma^+ = \frac{T - T_o}{f_o L / k_o} \tag{18c}$$

where C_o and k_o are nominal values of volumetric heat capacity and thermal conductivity at $T=0$℃, respectively. σ^+ is the dimensionless standard deviation of temperature readings. Besides, f_o and f^+ are a nominal and a dimensionless heat flux, respectively.

3.2 Test conditions

To examine the applicability of the proposed method to a nonlinear problem, the following temperature dependent properties of a virtual fluid are considered for test cases.

$$k = 1 + 0.01T,\ (\mathrm{W/m℃}) \tag{19a}$$

$$C = 1 + 0.001T^2,\ (\mathrm{J/m^3℃}) \tag{19b}$$

The test cases have been performed using artificially generated measurement data with and without errors. The errors are artificially embedded to the exact data by the following manner.

$$Y_i = Y_{exact,i} + \sigma r_i,\ i = 1, \cdots, N \times M \tag{20}$$

where r_i is a normally distributed random variable with zero mean and unit standard deviation within 99% confidence interval, i.e. $-2.576 < r_i < 2.576$. The random variable is generated by the IMSL® C function *random_normal*. $Y_{exact,i}$ and Y_i are the exact and noisy measured temperature, respectively. The numerical solution of the corresponding direct problem provides the exact data.

A parametric study is preformed for varying flow rate (Pe_H), error level (σ), and geometry (L/H). Besides, the number of future time steps is 5 for the test cases if not specified. In this inverse problem, the Peclet number as well as the dimensionless time step ($\Delta t^+ = k_o \Delta t C_o^{-1} L^{-2}$) affects the stability since parabolic nature of the partial differential equation is augmented by the convective term. The number of nodal points in each direction N is set equal to 40. The time step is set as $\Delta t = 0.1$s ($\Delta t^+ = 0.1$) and the estimation is conducted for a total of 50 time steps ($M=50$). Besides, the length is fixed as $L=1$m and the aspect ratio is given by $H/L=1$ if not specified.

3.3 Time-wise constant heat flux case

The spatial distribution of heat flux is given by the following expression.

$$f(x^+, t^+)/f_o = u(0.5 - x^+) \tag{21}$$

where $f_o = 100\mathrm{W/m^2}$, u is the unit step function. Figure 2 shows the reconstructed heat flux for varying Peclet number using the exact measurement data. For the decreased Peclet number ($Pe_H = 0.2$), the overshoot near the step is exag-

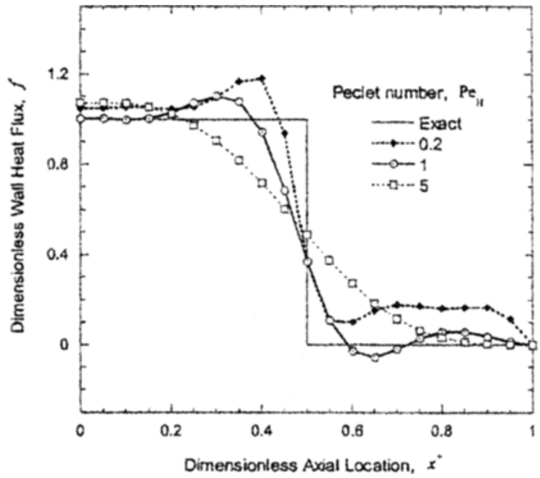

Fig. 2 Estimation of wall heat flux using exact measurement data for varying Peclet number (Pe_H=0.2, 1,5)

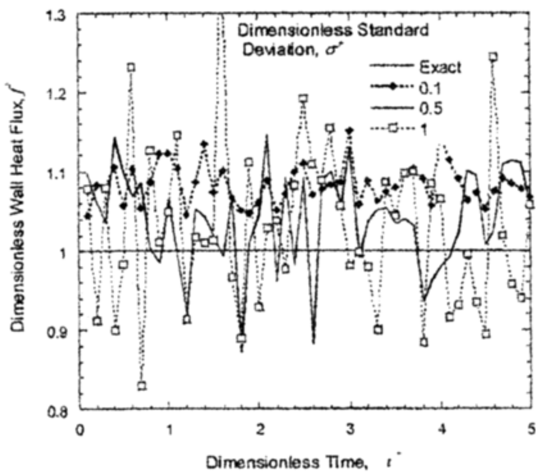

Fig. 4 Time-wise estimation of constant wall heat flux using noisy measurement data (σ^+=0.1, 0.5, 1) with m=5 and Pe_H=1 at x^+=0.25

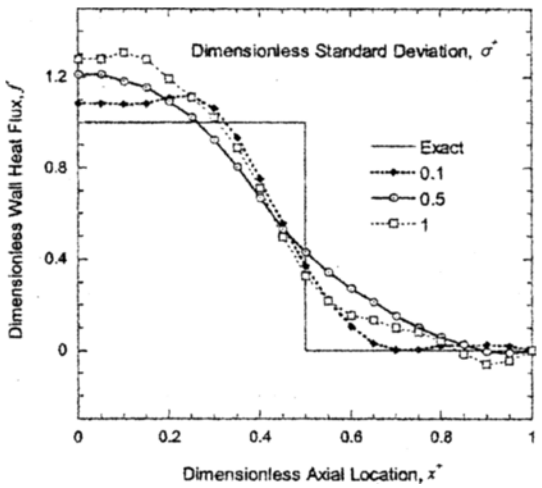

Fig. 3 Estimation of wall heat flux using noisy measurement data (σ^+=0.1, 0.5, 1) with m= 5 and Pe_H=1

gerated. On the contrary, for the increased Peclet number (Pe_H=5), a considerable loss of resolution is observed as can be seen in the figure. Thus, the estimation is expected to be inaccurate for the Peclet number out of a certain range. Figure 3 shows the result using noisy data for Pe_H=1 at t^+=2.5. This result demonstrates that the increase in error level leads to the loss in resolving power.

Furthermore, the heat flux value deviates considerably from the exact value in the upper step as can be seen in the figure. However, what is noticeable is that in spite of the increased error level, nearly no fluctuation is observed in the estimation. Contrarily, the reconstruction of the time varying heat flux shown in Fig. 4 shows significant disturbance at x^+=0.25. This evidences that the noise in measurement data incurs time-wise fluctuation and space-wise loss of resolution at the same time.

3.4 Time-wise triangular heat flux form

In order to investigate time-varying heat flux, triangular heat flux form is selected, which is expressed as

$$f(x^+, t^+)/f_o=(1-|2t^+/t_f^+-1|)\,u(0.5-x^+) \quad (22)$$

Other conditions are identical to the previous case. Estimation is performed using noisy measurement data, whose standard deviations are given by σ^+=0.02, 0.1 and 0.5. Here, σ^+=0.1 approximately corresponds to 0.5% error based on the average of measurement data.

Figure 5 shows the estimated heat flux at x^+= 0.25. The fluctuation becomes more intense along as the error level is increased. However, even with σ^+=0.5, a stable estimation is achieved as can be seen in the figure. Figure 6 shows the result with varying the number of future time steps. As the number of the future time steps increases, the heat

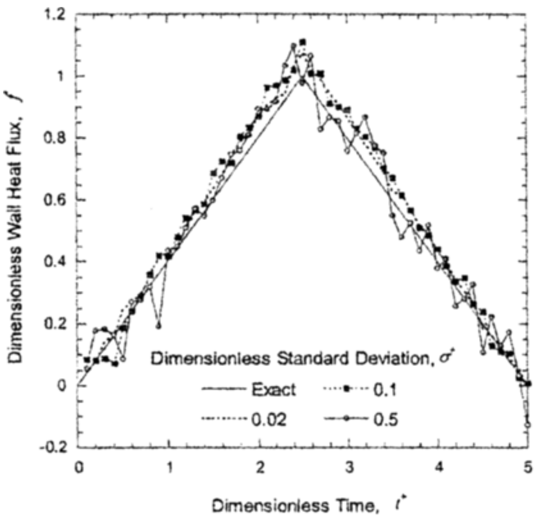

Fig. 5 Time-wise estimation of triangular wall heat flux using noisy measurement data ($\sigma^+=0.02$, 0.1, 0.5) with $m=5$ and $Pe_H=1$ at $x^+=0.25$

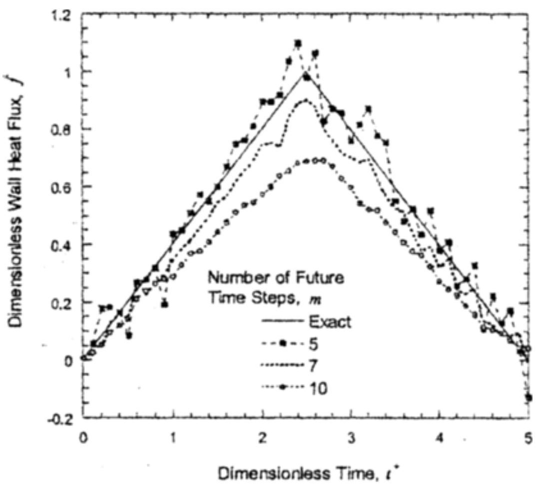

Fig. 6 Time-wise estimation of constant wall heat flux using noisy measurement data ($\sigma^+=0.5$) for varying future time steps ($m=5$, 7, 10) with $Pe_H=1$ at $x^+=0.25$

flux becomes smoother. For $m=10$, the heat flux shows a considerable bias near $t^+=2.5$. Figure 7 shows the heat flux form at $t^+=2.5$ recovered for different aspect ratio ($H/L=0.1$, 0.5, 1, 2, 10). As the aspect ratio increases, the duration of heating decreases. As a result, increased bias is observed for $H/L=2$. The peak point on the upper side near the step moves backwards along

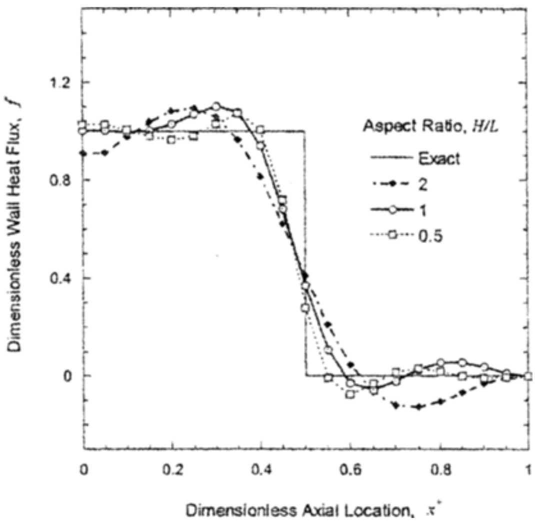

Fig. 7 Time-wise estimation using exact measurement data for varying aspect ratio ($H/L=0.1$, 0.5, 1, 2, 10) with $m=5$ and $Pe_H=1$ at $t^+=2.5$

with increasing aspect ratio as can be seen in the figure. It is evident that the measurement data is very insensitive to surface heat flux when the aspect ratio is reduced. On the other hand, when the aspect ratio is increased, the estimation requires more iteration since the increased temperature variation due to the longer heating causes higher nonlinearity.

3.5 Singularity issue

Figure 8 shows a computational result using the conventional and sequential methods for a case, which considers the following heat flux form.

$$f(x^+, t^+)/f_o=u(t^+-0.5) \tag{23}$$

The result shows that the singularity is resolved with the sequential method. Here, the implementation of the conventional method (the whole domain method) is accomplished by following a previous work (Huang and Özişik, 1992).

Finally, the iterative improvement technique is tested with the following heat flux form.

$$f(x^+, t^+)/f_o=u(x^+-0.5) \tag{24}$$

Figure 9 compares the results for three different cases: (1) with the iterative improvement and unknown $f(L, t^+)$; (2) without the iterative improvement and unknown $f(L, t^+)$; (3) without the iterative improvement and known $f(L, t^+)$. As can be seen in the figure, the result for case (2) is unacceptable while the results for cases (1) and (3) are comparable. This result verifies effectiveness of the iterative improvement. Here, the calculation for case (1) is conducted with $p=2$ and $R=5$.

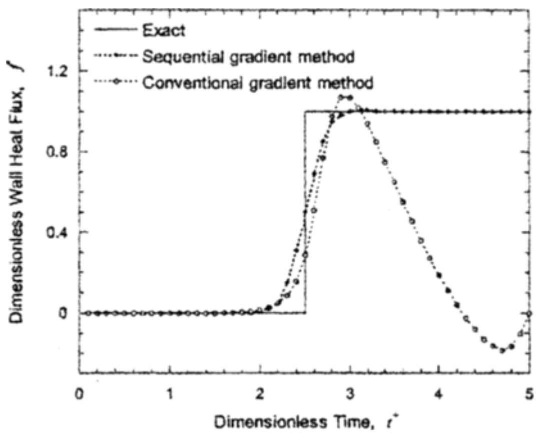

Fig. 8 Time-wise estimation of constant wall heat flux using exact measurement data ($\sigma^+=0$) with $Pe_H=1$ at $x^+=0.25$. Comparison of the sequential gradient method ($m=5$) and conventional gradient method

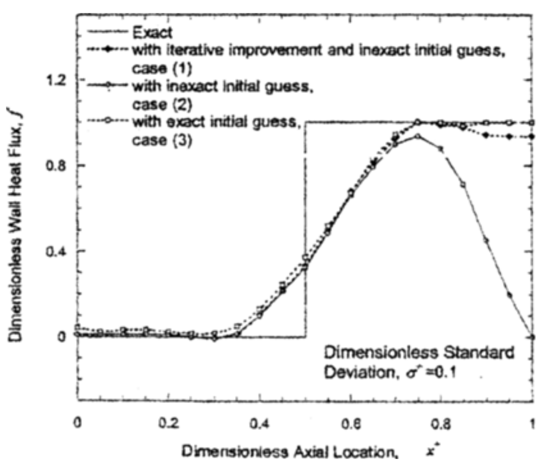

Fig. 9 Estimation of wall heat flux using noisy measurement data ($\sigma^+=0.1$) with $m=5$ and $Pe_H=1$. Comparison of results with and without iterative improvement, and with exact and inexact initial guesses

4. Conclusion

The sequential gradient method is applied to an inverse convection problem. The problem considers a laminar convective flow between parallel plates. The upper plate is exposed to unknown heat flux while the lower plate is insulated. The unknown heat flux is estimated inversely from temperature measured on the lower plate. The function specification method is incorporated to stabilize the sequential estimation. The relevant adjoint formulation has been made. Accuracy and stability have been examined extensively for various test cases. Furthermore, the proposed method is beneficial in resolving the nonlinearity caused by the temperature dependent thermophysical properties and the singularity in the final time thanks to the sequential estimation. Besides, it is observed the singularity in the space domain can be alleviated with the iterative improvement technique. The overall performance of the method is verified valid from several stringent tests as shown in the results.

Acknowledgments

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